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REPORT No. 4

On the Mechanics of the Tornado

NATIONAL SEVERE STORMS PROJECT

U. S. Department of Commerce
Weather Bureau
WASHINGTON, D. C.
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No. 1. National Severe Storms Project Objectives and Basic Design. Staff, NSSP, March 1961.

No. 2. The Development of Aircraft Investigations of Squall Lines From 1956-1960. B. B. Goddard, February 1962.

No. 3. Instability Lines and Their Environments as Shown by Aircraft Soundings and Quasi-Horizontal Traverses. D. T. Williams, January 1962.

U. S. DEPARTMENT OF COMMERCE
Luther H. Hodges, Secretary
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by
J. R. Fulks
U. S. Weather Bureau, Chicago, Ill.

Washington, D. C.
February 1962

FOREWORD

This discussion of the mechanics of tornadoes is published in the NSSP preprint series as a means of making the ideas expressed generally known to the profession. Observational evidence gathered by the project will undoubtedly either verify or modify portions of the hypothesis.

This paper, by an author who has contributed significantly to the study of severe convective phenomena, is presented for the purpose of stimulating thinking in an area which, despite its importance, has been little explored.

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ON THE MECHANICS OF THE TORNADO*

J. R. Fulks

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ABSTRACT

A model is developed which describes the cause and mechanism of the severe tornado. The suggested process is essentially as follows:

(1) An elongated or squall-line type thunderstorm forms in a region where the winds veer with height.

(2) A vertical cyclonic vortex of several kilometers radius develops around the right-hand end of the thunderstorm, caused by vertical exchange of momentum. The exchange of momentum produces both cyclonic vorticity and upper divergence in that part of the thunderstorm. Other factors involved in producing the initial vortex, according to this model, are the separation of uplifted air from the updraft as the storm moves, subsidence of cold air at low levels, and adiabatic warming aloft in the lower stratosphere.

(3) An updraft through the vortex begins at some point near the ground where the initial vortex extends into warm surface air. Ground friction is an important factor in producing the necessary low-level convergence.

(4) Buoyancy of the rising air causes it to be stretched vertically, and therefore to converge horizontally, so as to increase the cyclonic rotation. Continued passage of buoyant air upward through the vortex causes a cumulative increase of kinetic energy in the vortex until there is a fully-developed tornado. On the basis of quantitative estimates, all the features of the proposed mechanism seem to be of the proper order of magnitude.

1. INTRODUCTION

Our understanding of the mechanics of tornadoes and waterspouts is still quite meager even though their cause has been a subject of long speculation. Part of the problem is to explain their association with the thunderstorm, though the fact of this association seems well established, at least for the severe tornado. Ferrell (1889) followed a thermodynamic approach and classified the tornado as a thunderstorm, but his model does not appear to require pre-existence of a thunderstorm. Bigelow (1907,

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*Presented at the 133d National Meeting of the American Meteorological Society, Miami Beach, Florida, November 17, 1954. Minor revisions made September 1961.

1908) made extensive computations of the wind structure and other features of a large waterspout, and a tornado, based on vortex theory, but his work does not explain how the conditions originate nor why they are associated with thunderstorms.

An important contribution came from Wegener (1917, 1918,1928) who suggested that the tornado represented the downward extension to the ground of a horizontal vortex across the forward part of the thunderstorm. Wegener's ideas did not receive great support in this country, perhaps because of questions on the relative scale of the tornado as compared to the horizontal vortex, and on the manner by which energy could be concentrated in the vertical portion of such a vortex. However, the necessary existence in the thunderstorm of vortex lines in approximately the manner postulated by Wegener suggests that his ideas must apply to the problem; also recent data (especially Huff, Hiser and Bigler, 1954; and some other radar data) seem to suggest that the tornado occurs near the right-hand end of the thunderstorm as required for a cyclonic vortex according to Wegener's theory.

In recent years, synoptic meteorologists (Lloyd, 1942; Showalter, 1943; Fulks, 1943, 1951; E. M. Brooks, 1951; Fawbush, Miller and Starrett, 1951) have learned to associate certain parameters with tornado occurrence. The tornado seems to develop with an elongated thunderstorm, and most frequently along the instability line. Some of the conditions known to be generally associated with tornado-producing thunderstorms are (a) veering of wind with height in the lower troposphere (corresponding to geostrophic warm advection), (b) existence of a supply of warm moist air in the lower 1 to 3 kilometers, (c) marked conditional and convective instability, and usually (d) dry air above the moist layer. From synoptic experience alone we cannot say which of these parameters are merely associated with the squall line and which, if any, are involved directly in the mechanics of the tornado. A plausible theory of the mechanics of the tornado and the associated thunderstorm activity should not only indicate the probable role and importance of the different parameters, but it will also serve as a useful guide to the type of observations needed to supply further specific information on the nature of tornadoes.

2. REMARKS ON THE MECHANICS OF THE ACCOMPANYING THUNDERSTORM

Newton (1950) pointed out that ". . . . .An essential source of energy for maintaining the squall line activity appears to be the kinetic energy of air brought down from higher levels." Harrison and Orendorff (1941) had earlier shown that squall lines tend to be accompanied by a pseudo-cold front caused by evaporation of rain into dryer air. The temperature and moisture content of this colder air at the ground frequently suggests that it has descended from heights of 3 or 4 kilometers, assuming descent at the saturated adiabatic lapse rate; and from even greater heights if account is taken of mixing with the environment and of time required for evaporation. Probably to some extent there is downward motion in the thunderstorm from the drag effect of falling rain (C. F. Brooks, 1922; Byers and Braham, 1949), but evaporative cooling seems to be far more important because in at least the tornado-type of thunderstorm the vertical distribution of wet-bulb temperature in the surrounding air is such that evaporatively-cooled air will sink, and if kept saturated or nearly so it will accelerate in its downward motion all the way to the ground. Normand (1946) pointed out that this down-current within the thunderstorm represents an appreciable part of its transformation of potential into kinetic energy.

Examination of a typical dry-over-moist sounding in the region of tornadoes, such as the Fawbush-Miller (1952) mean tornado sounding, will show that the amount of moisture required for saturating the dry air may be of the same order of magnitude as the total amount of moisture that can be precipitated from the lower moist air. This is figured on the assumption that all the lower moist air will be lifted to very high levels and that all the dry air below, say, 5 or 6 kilometers over the same region, will become saturated and remain saturated while it sinks a sufficient amount to displace the moist air. In fact the amount may often be insufficient, as was pointed out by Braham (1952), though in typical thunderstorm conditions there is low-level convergent flow which provides greater total moisture supply to the thunderstorm than is indicated by a vertical sounding taken ahead of the thunderstorm.

Once a pattern of sinking rain-cooled air has been established, there is a rather sharp boundary within the thunderstorm between descending cold air and ascending warmer air, as was shown for example by Byers and Braham (1948) on the basis of Thunderstorm Project data. This boundary is illustrated schematically in figure 1. The sloping density discontinuity might be called the "thunderstorm front," though it has a much steeper slope and is distinct from any cold, occluded, or warm front surface. The sinking cold air will have downward momentum mainly produced by the excess of density over its surroundings, and in addition a horizontal component carried down with it from above.

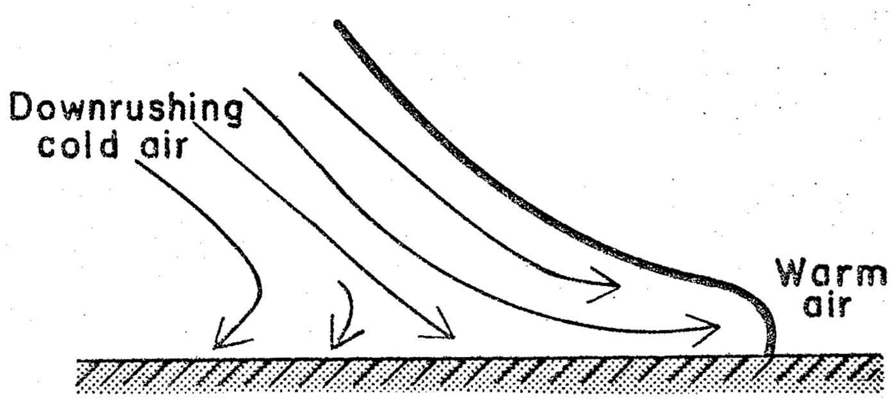

Figure 1.— "Thunderstorm front", and the pattern of sinking rain-cooled air.

The outspreading cold air on the forward side of the storm will not only lift the warm air ahead of it, but will also impart som forward motion to the warm air consistent with the forward movement of the thunderstorm.

It is estimated that the upward thrust which may be imparte to the rising air by the exchange of momentum between levels in the tornado type thunderstorm is of the same order of magnitude, though generally not quite as great, as the effect of thermal in stability. If we assume the thunderstorm front to make an angle of 45° to the horizontal and to have a horizontal speed of 30 m sec., with respect to movement of the warm air in the same direc tion, then assuming no horizontal movement imparted to the warm air, the warm air would acquire an upward motion of 30 m./sec.; actually this is an overestimate because the warm air will also acquire forward motion. For the Buoyancy effect, if we assume temperature in the rising air column to be everywhere 5°C. warme than the environment, we obtain for the vertical acceleration, $dw/dt = g(\Delta T/T) \approx (5 \times 10^3)/250 = 20$ cm./sec.^{2} Since $w = (2\ dw/dt)^{1/2} z^{1/2}$ for constant acceleration we compute for the top of a 10 km. vertical column a resulting vertical speed of 63 m./sec or an average with respect to height ($= 2/3\ w_z$) of 42 m./sec. These examples are rather extreme but not impossible. Actual speeds are of course reduced by frictional and mixing effects. The main point is that the buoyancy of the rising air and the negative buoyancy of the sinking air are both important to the energy of the thunderstorm, in accordance with what was pointed out by Normand (1946).

A consequence of strong vertical motion in the tornado type thunderstorm, of which the veering of environmental winds with height is known to be an accompanying feature, is that rising air will reach the upper levels still retaining a portion of the horizontal momentum which it had at lower levels. Factors tending to change the momentum can have only a partial effect because of the time required to overcome inertia. It is evident that, in general, the greater the vertical speeds the greater the tendency for the rising air to retain its original horizontal momentum. In the typical tornado situation, where for example, the warm moist air may be moving from the south and the air at 3-4 km. from the west, the lower air upon reaching higher levels must have a component of horizontal motion that is upstream with respect to the surrounding air.

3. RELATION OF THE TORNADO TO THE THUNDERSTORM

Markgraf (1928) seems to have supplied a link in the understanding of the tornado when he pointed out that vortices with vertical axes should develop near the two ends on the forward side of the thunderstorm because of relative upstream movement of the convective cloud. His ideas, submitted as a comment on Alfred Wegener's (1928) mechanical theory, supplemented Wegener' hypothesis of a horizontal "mother-whirl" aloft along the forward side of the thunderstorm. It is perhaps of some interest that data of the Thunderstorm Project (Byers and Braham, 1949) verify Markgraf's basic assumption that the thunderstorm cloud often has a component of motion upstream with respect to the surrounding air.

While the ensuing discussion is not based explicitly on the work of Wegener and Markgraf, it accepts Wegner's theory of a vortex, within which tornadoes occur, extending from the ground upward and thence in part or perhaps mostly horizontally aloft across the upper levels of the thunderstorm. It will not be assumed, however, that all the vortex lines extend across the forward part of the storm. For reasons to be discussed later, a part of the vortex lines should extend upward, probably into the stratosphere, though these need not coincide with lines of flow. The diameter of the vertical vortex is estimated to be several kilometers, say, roughly ten times the diameter of the individual tornado. This vortex should reach the ground somewhere near the right-hand end of the accompanying thunderstorm, it may also on occasions reach the ground near the left end where divergence patterns do not favor the development of tornadoes.

The direction of rotation of the vortex where it reaches the ground on the right side of the thunderstorm, according to this model, is cyclonic; if it should reach the ground on the left side its rotation would be anticyclonic. Its contact with the ground is taken to be the tornado low as discussed by E. M. Brooks (1949). A vertical section of the model is illustrated schematically in figure 2. Only a portion of the vortex can ever be visible to an observer on the ground; it is manifested in part by a rotating

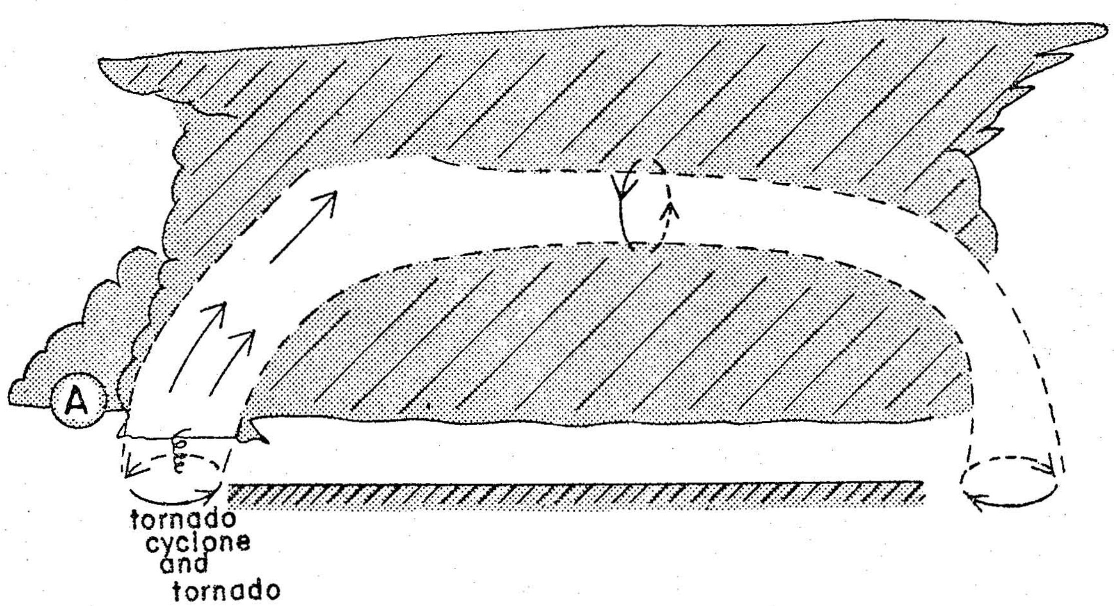

Figure 2.— Schematic vertical section across the forward portion of a tornado thunderstorm. The rotating "mother cloud" is located at "A". Probably only a portion of the vortex lines from the tornado region extend horizontally across the forward portion of the storm, others extending into the upper middle part of the thunderstorm, also upward into levels above the cloud top. The portion of the vortex region indicated in the diagram as being surrounded by cloud is mostly inside the cloud.

cloud such as is shown at (A) in figure 2; this r tating cloud has been observed and reported on many occasions. The tornado itself is a more concentrated vortex. For it to be observed, visible particles (water or dust) must be present, and there must also be continuous convergence throughout the visible column to prevent dispersal of the visible particles as has recently been shown by Kangieser (1954).

4. RELATIVE MOTION AND CIRCULATIONS OF THE THUNDERSTORM

When there are west winds in middle levels and south winds in the lower levels, corresponding to conditions frequently observed in tornado regions, we may expect a cloud direction about as shown by vectors V_C in figure 3, and relative motion of the cloud with respect to its surroundings such as vectors R_U and R_L

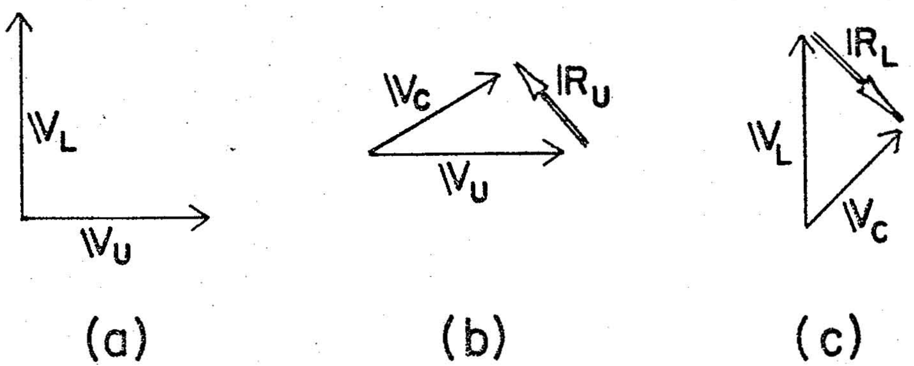

Figure 3.— Wind vectors: V_L is the lower-level wind, V_U the middle- or upper-level wind, V_C the assumed cloud movement vector, R_U ($=V_U-V_C$) the movement of the cloud at upper levels relative to surrounding air at the same level, and R_L the corresponding vector for low levels. V_C corresponds nearly to the vectorial mean of the lower- and upper-level winds, but is biased toward the ambient wind at the particular level concerned.

in the same figure. There will be a wake on the side of the cloud opposite to its relative direction of movement, as shown in figure 4, for middle and upper levels. This corresponds to the region where strong turbulence is sometimes observed in the clouds preceding a thunderstorm. These turbulent motions are disorganized whirls not associated with tornadoes; tornadoes will occur only in the organized whirl shown at (A) in figure 4 if the suggested model is correct. The initiation of an organized vortex, sufficiently deep and sufficiently isolated so as not to be easily destroyed by mixing and interaction with whirls of opposite rotation, evidently requires more than the turbulent conditions in the wake of the cloud. The flow of air past the cloud provides for a stronger more stable vortex at (A) than anywhere else in the wake except for a similar vortex of opposite rotation at the other end of the cloud.

Before proceeding further, it is useful to note the principal vortex regions in a thunderstorm as they may be observed in a vertical cross section (fig. 5). In figure 5 it is assumed

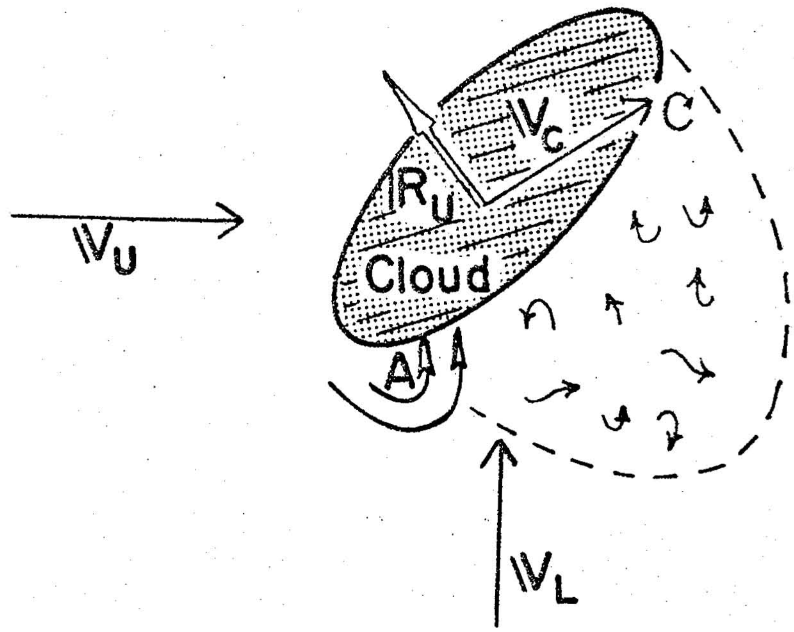

Figure 4.— Schematic horizontal section of cloud, showing position of wake as related to relative cloud movement. Vectors as in figure 3. The tornado vortex is at "A". Thin arrows show streamlines which, along with relative dimensions, are hypothetical. Curved streamlines in the wake, except near the ends of the cloud, are intended only to show random turbulent motions.

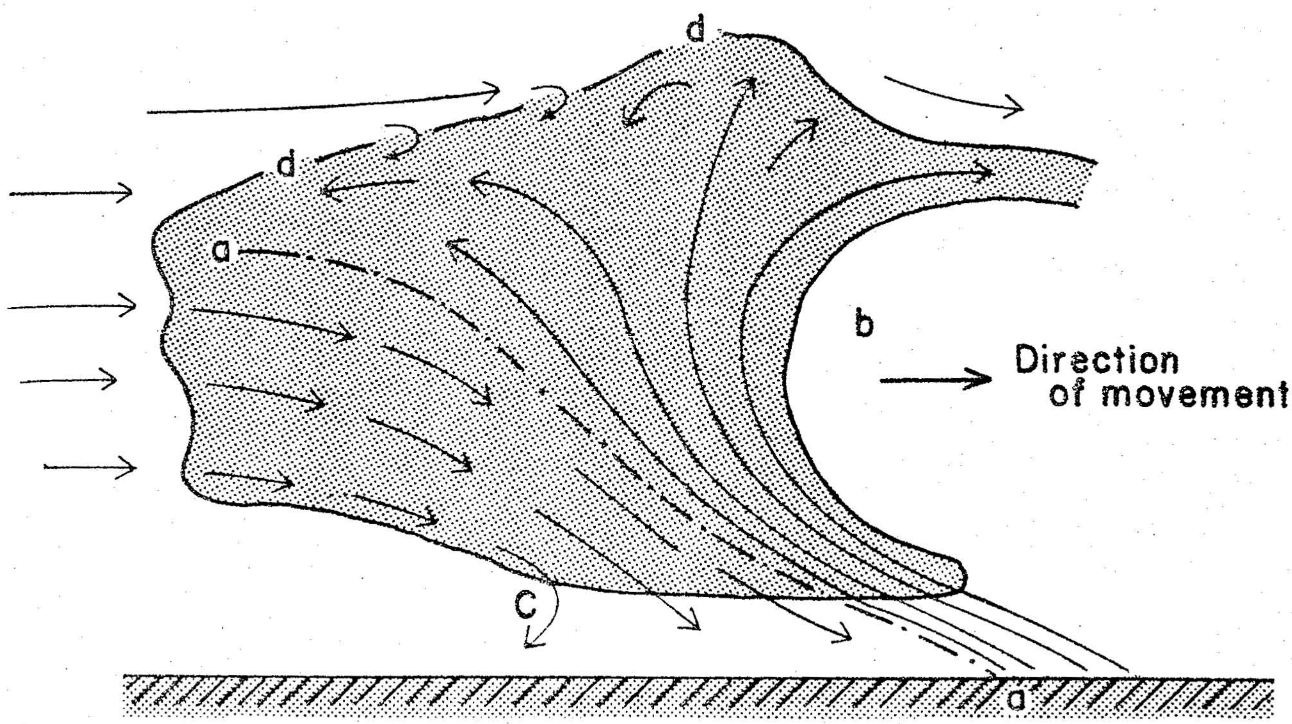

Figure 5.— Idealized vertical section through a thunderstorm showing the principal zones for which the vortex lines are mainly horizontal. Vorticity, as viewed from the right-hand end of the thunderstorm, is positive along aa, and negative at b, c, and along dd.

that the environmental winds veer with height. Most of the features of this diagram are based on observed conditions though some details such as horizontal diffluence in the downward flow near the ground and the shear along dd are highly probable conditions based on indirect inferences. The shear zone between sinking rain-cooled air and the uprushing warm air is shown at aa; this shear zone is thermally stable and is not likely to develop rotation except possibly on a small scale. Its shear is in the wrong sense to be connected to a cyclonic vortex at the right-hand end of the thunderstorm, but a strong concentration of vorticity along aa is important in permitting a large amount of vorticity of opposite sense to develop elsewhere in the storm. The vortex at b is produced by the upward and outward flow of warm air, and as already stated it probably connects to the tornado vortex. At c is a vortex, rotating in the same sense as the vortex at b, produced by low-level outflow from the region of high surface pressure associated with the downdraft; it has no likely direct connection to the tornado vortex because it is in the cold air.

It is the writer's opinion that of the shear zones extending horizontally through the thunderstorm, the one at dd is probably most important to the tornado, though it undoubtedly acts in conjunction with b where the rotation is in the same sense. Vortex lines extending from the low-level tornado vortex upward and into regions dd and b probably for the most part extend downward to the ground on the left end of the thunderstorm where, however, no reason is found for their concentration. An alternative possibility is that at least a portion of the vortex lines connect with the region aa at the left end of the storm and thence across it to the ground in the cold air at the right-hand end of the storm. Some vortex lines may turn downward into cold air to the rear of the surface tornado because, as will be discussed later, the air which goes aloft apparently in part displaces sinking cold air in that region. The magnitude of the shear at dd is in part a function of the increase of wind speed with height in the upper troposphere.

A postulated horizontal cross section through an upper level of the thunderstorm is shown in figure 6b. The advance of the cold wedge (fig. 6a) requires that new streamlines of upward rising warm air be constantly formed at low levels immediately ahead of the forward edge of the cold air. Evidently, as new upward streamlines develop, the cloud mass will tend to grow on its forward side, and the storm as a whole will move in part by development. In the interior of the thunderstorm, perhaps well toward the rear, there should be some level where the cold wedge must become horizontal or nearly so, especially when the horizontal cross section of the storm is large. In that part of the storm the uplifted air should tend to be cut off from its warm air source, so that air from below can no longer be fed into that part of the storm.

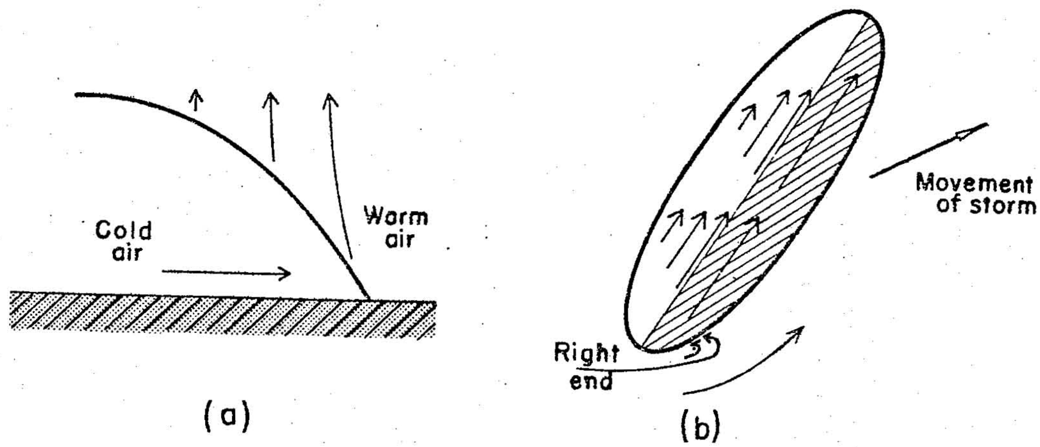

Figure 6.— (a) Vertical section showing development of new streamline components of forced upward motion associated with advance of cold wedge, and their diminution as the wedge becomes horizontal (see text).
(b) Horizontal slice through upper portion of thunderstorm, the active updraft part of which is indicated by hatching. Arrows in cloud area show assumed components of horizontal wind in the direction parallel to the long axis of the cloud. Toward rear of cloud, air has been in cloud longest and this component has diminished (see text).

5. DEVELOPMENT OF A FIELD OF DIVERGENCE ALOFT AND ITS ASSOCIATED VORTEX

The shaded portion of figure 6b represents the part of the lifted air that is still being fed from below. This air will carry with it an appreciable component of horizontal momentum parallel to the axis of the convective cloud, and because the differing direction of environmental air-flow past the right-hand end of the cloud is such as to require horizontal divergence of the wind, there will be a consequent tendency toward development of lower pressure at the right-hand end of the storm. But so long as there is an active updraft from below, the tendency toward a net divergence, and therefore of pressure fall, is likely to be compensated by vertical convergence. There may, however, be some mixing between the uplifted and the environmental air, an effect that must tend to form a cyclonically curved streamline around the right forward corner of the thunderstorm, because the mixed air will tend to acquire the momentum of both air masses in such a way as to produce cyclonic rotation.

Once the upward streamlines of warm air from below are cut off (unshaded portion of the cloud in figure 6b), the divergent horizontal wind field along the right-hand edge of the storm must cause a net mass divergence, and therefore a lowering of pressure in that region at upper levels of the thundercloud unless there is compensating vertical inflow from other sources. The possible sources are upward-moving cold air from below and downward-moving potentially warmer air from above. Under the conditions postulated, there must be cold air below the level of incipient pressure fall because the existence of the cold air is taken to be an important factor in causing the cut-off of upward-moving warm air in the rear (or perhaps also the middle part) of the thunderstorm (cf. fig. 6a).

Also, it seems certain that the cloud mass aloft will tend to move away from the lower cold air in the vicinity of the right-hand end of the storm, because the cloud will have a component of movement parallel to the long axis of the storm and away from its right-hand end, whereas the cold air at the ground is known from observation to have in general little, if any, component of motion in that direction. Because the cold air below the region of incipient pressure fall is relatively dense, and tends to sink, it must tend to contribute toward net three-dimensional divergence aloft and therefore toward further pressure fall aloft.

Because the lapse rate is generally less than the adiabatic, especially above the tropopause, downward motion will be accompanied by local warming. Buoyancy of the locally warm air will tend to prevent its further downward motion, and will therefore oppose the tendency of the downward motion to contribute to net convergence, and coincidentally the warming aloft will contribute to pressure fall. Therefore, <u>around the right-hand end of the thunderstorm, in cases where the environmental winds veer with height, there must generally be a region of net divergence and pressure deficit centered at the upper or middle levels of the thunderstorm</u>.

<u>The strength of this pressure deficit should be a function of the amount of veering, the strength of the environmental winds at both upper and lower levels, the strength of the upward flow, the amount of temperature contrast between the rain-cooled air and the surrounding air, the heights to which convection extends, and the size of the storm (partly for reasons to be discussed later); possibly also the variation of environmental wind speed with height in the upper troposphere, the depth of the rain-cooled air, and the lapse rate of the environment at the higher levels of the thunderstorm</u>. Not all these factors are independent.

At the left-hand end of the thunderstorm (fig. 6b) the convergent flow should tend to increase the pressure aloft. Any local increase of pressure of this nature will be superimposed upon the lower levels and thus cause horizontal divergence in the low-level air. While an upper vortex must tend to develop mechanically aloft at the left-hand end (fig. 6b), the lack of surface convergence should prevent any intensification of it. <u>There is therefore no reason to expect tornadoes to develop at the left-hand end of the storm</u>.

On the assumption that the horizontal field of lowered pressure aloft at the right-hand end of the thunderstorm is roughly circular or elliptical it may be judged that its diameter or major dimension, assuming it to be approximately half the width of the thunderstorm, is of the order of a few kilometers up to perhaps as much as 10-20 km. in some cases. The development of such a center of low pressure is consistent only with simultaneous development of vortex motion. The reason is that, on this scale of phenomena, the effect of the earth's rotation is unimportant so that a horizontal field of low pressure can only be balanced by the centrifugal force and the accelerations; at some equilibrium stage the accelerations must cease and at that stage the horizontal rotation must have already developed if the local pressure deficit continues. Since at the right-hand end of the storm the pre-existing vorticity is cyclonic, the direction of rotation of this vortex must be cyclonic.

An estimate may be made of the order of magnitude of the total depression of pressure aloft by the mechanism so far discussed. As illustrated in figure 6b, the air within the cloud has a component of motion away from the right-hand end of the storm, a motion that is opposed by the inertia of the air flowing around the right-hand side and by the vertical stability of the air above and below, as discussed previously. In order to maintain continuity of mass, neglecting the small effect of net decrease of pressure or density, the air within the cloud must decelerate as it is cut off from its low-level source. We may compute the pressure field associated with the deceleration provided the rate of deceleration and total length of the cloud mass can be estimated.

Referring to figure 6b, suppose the thunderstorm moves forward at a speed of 20 m./sec., and that the air reaching the upper levels of the thunderstorm on its forward side has a horizontal component of speed of 25 m./sec.; suppose further that the width of the storm is 20 km. but that the total deceleration takes place through a distance of 10 km. from the middle to the rear of the storm. The time required for the storm to move 10 km. is 10 x 1000/20 or 500 sec. The total deceleration of horizontal speed is therefore from 25 m./sec. to zero in 500 sec., or at a rate of 5 cm./sec.^{2} Suppose the length of the cloud to be 40 km., and the deceleration to be uniform throughout its length. Taking air density as 0.7×10^{-3} gm./cm.^{3} (approximate value for the 500 mb. level), we obtain from the equation of motion, with Δs = cloud length,

$$\Delta p = \rho\left(\frac{dv}{dt}\right)(\Delta s) = 0.7 \times 10^{-3} \times 5 \times 40 \times 10^{5} \text{ dynes cm.}^{-2}$$

$$= 14 \text{ mb.},$$

where Δp is the total computed difference in pressure between the right and left ends of the cloud.

The net effect, considering the average pressure through the length of the cloud to remain the same, will be an increase of pressure by 7 mb. on the left-hand end and a decrease of 7 mb. on the right-hand end. Probably this is greater than the true effect in most cases, but the computation shows the likely order of magnitude as well as the probable importance, to tornado development, of the length of the convective cloud. In speaking of the length of the convective cloud, no account is taken of the fact that a single elongated thunderstorm may be composed of numerous updraft cells, but their combined effect is considered.

According to this model the lowest pressure aloft will be along the line, around the right-hand end of the storm, separating the uplifted air from the environmental air. This requires a horizontal pressure gradient to develop also in the environmental air; its strength cannot be specified, but a reasonable order of magnitude is 1 mb./km., sufficient to cause the environmental air to turn sharply to its left and form rotary motion. Development of the low pressure, according to this model, is of course also a function of the strength of the environmental wind, or more exactly, the vector difference in velocities of the environmental and uplifted air. If the strength of the environmental wind is too slight, it will readily turn into the low pressure center and tend to cause it to fill; whereas the inertia of the strong wind, acting in a direction different from the uplifted air (so as to strengthen the horizontal divergence field) will cause rotary motion and permit a greater total reduction in pressure.

At the rear of the cloud, the horizontal pressure gradient established within the cloud may even be effective in reversing the component of horizontal air motion parallel to the long axis of the cloud, especially near the incipient vortex, so as to form the rear portion of the vortex; the net result being a winding up and perhaps mixing of the cloud and environmental air in the vortex, the vortex then becoming largely if not entirely engulfed in cloud. The model so far presented is one of a vortex, on a larger scale but much less intense than the tornado, formed entirely by inertial effects resulting from the exchange of momentum between levels. Later, on the basis of vortex theory, it will be shown that the order of magnitude of the pressure deficit as computed above agrees with the magnitude of the depression of pressure in a vortex of the size postulated and having a maximum linear speed of rotation similar to the environmental wind speed.

<u>The formation of this inertial vortex aloft is proposed as a necessary and primary cause of tornadoes.</u> It is not suggested that its existence always results in tornadoes; on the contrary it probably exists in most if not all thunderstorms having the requisite veering of winds with height, though only a small percentage of such thunderstorms are accompanied by tornadoes. But its strength will vary greatly for reasons suggested above. Undoubtedly, its strength is a factor in whether or not tornadoes will occur, but since the tornado is a phenomenon having much greater kinetic energy than the inertial vortex it seems evident that there must be additional criteria for the actual occurrence of tornadoes, especially the availability of additional energy and means of utilizing it.

However, the concept of the inertial vortex not only provides for an organized vortex within which it is postulated that tornadoes develop, but if upward motion should develop within the vortex, this model provides for rapid removal of mass aloft because of the large amount of horizontal divergence that is made possible by the tendency of the cloud mass to move away from the vortex. However, it should be noted that the model requires a core of warmer air (caused by subsidence) at still higher levels and therefore a low-pressure center diminishing upward with height. Corresponding to this low pressure, there must also be rotary motion decreasing with height and therefore probably an extension upward (without corresponding upward motion) of some of the vortex lines which must then tend to spread outward and become horizontal at the higher levels.

For this low pressure aloft to be effective in producing a center of low pressure at the ground, its edge must in general extend outward in some direction beyond the low-level boundary of the cold air Actually, the pressure field of the upper inertial vortex would superimpose itself upon the lower levels and must in some degree affect the movement of the surface cold air so as to produce some form of an undulation along its boundary. Such undulations were found by Williams (1948) in his detailed surface analyses of instability lines passing over the micronetwork near Wilmington, Ohio.

The surface pressure within the cold air under the vortex would be lower than elsewhere in the same depth of cold air, so that the portion of the surface boundary of the cold air nearest the upper vortex should tend to recede toward the upper vortex. This might conceivably take place anywhere around the right-hand end of the thunderstorm Therefore, as a result of the superimposed lower pressure from above, an area of surface low pressure should tend to develop along the boundary of the cold air at the point nearest the upper vortex It is perhaps only in special cases that the cold air boundary will move under the upper vortex so as to permit the pressure to be lowered within the surface warm air. The extension of the upper low pressure downward into a region of warm surface air then makes possible the establishment of convection from the ground upward through the vortex, provided the low-level air is sufficiently warm and moist, a condition very likely to be met in thunderstorm areas.

The development of the surface low pressure center will cause horizontal low-level convergence, which in turn will result in rotation if the converging air possesses vorticity. Unless the direction of rotation of the rising air is the same as that of the vortex aloft into which it rises, the rising air will certainly tend to weaken the vortex aloft; therefore, it seems a necessary condition to further development of the vortex that the low-level air should possess cyclonic vorticity. There is likely to be cyclonic shear in the surface warm air around the right-hand end of the thunderstorm, because that region is to the left of the strong flow of surface warm air into the main part of the thunderstorm. It is possible also that the inflowin warm air may often have its cyclonic vorticity increased by flow ing around colder air (if the colder air is to its left) or around a topographic obstruction, such as a hill, and that such a additional source of vorticity may have a bearing on the likelihood or exact location of tornado development.

6. VERTICAL MOTION IN THE TORNADO

It is considered here that the only possible sustained vertical motion in the visible tornado is upward. In making this statement, the term "tornado" is defined only as the rope-like o inverted cone-shaped vortex extending downward to the ground fro the tornado cloud, and not to the cloud itself nor the region su rounding the visible tornado. Upward motion in the visible tornado is in agreement with observational evidence. Kangieser (1954) has shown that a component of radial inflow is required to retain water or dust particles in the vortex against the action of centrifugal force; such radial inflow in the lower level of the vortex which touches the ground is possible only with upward flow in the vortex. Near the ground, upward motion is necessitated by frictional flow into the low pressure region. And because the tornado low itself moves with respect to winds in the lower one or two kilometers, local convergence throughout th visible tornado is required by the resulting isallobaric field, the local pressure changes throughout the lower layer being largely superimposed from above.

The fact that vertical circulation in the visible tornado must be upward does not preclude passive downward movement of ai in the upper central portion of the relatively wide funnel-shape tornado (probably not the rope-like tornado); in fact, such subsidence with consequent warming in the center seems necessary if we are to account for the extremely low pressures and high winds that evidently occur in many tornadoes. Also, the requirement for upward motion does not preclude subsidence in the cold air ir rather close proximity to the tornado, especially on its rear sid provided the tornado is moving sufficiently fast to prevent the cold air from reaching and ascending in the vortex.

While inflow and upward motion must begin with formation of even the weak surface low-pressure center, an equilibrium with frictional dissipation will tend to be reached before a severe tornado develops unless there is some additional cumulative concentration of energy in the central portion of the vortex. The only apparent additional source of energy is the buoyancy of the rising air. Conditional and convective instability normally exist in thunderstorm regions, being necessary to the development of the thunderstorm, but for this energy to contribute to the tornado, the low-level portion of the vortex must extend into the warm moist air at the ground. It seems axiomatic that if the vortex is to receive further energy from buoyancy forces, the further development will take place where, within the confines of the original vortex, the additional energy is most readily available. The resulting vortex should therefore have some horizontal component to its orientation, probably in the main above the level of the lower moist air (above the visible tornado) thence above the lower cold air toward the center of the upper vortex. Figure 7 shows a picture of the suggested form of the vortex.

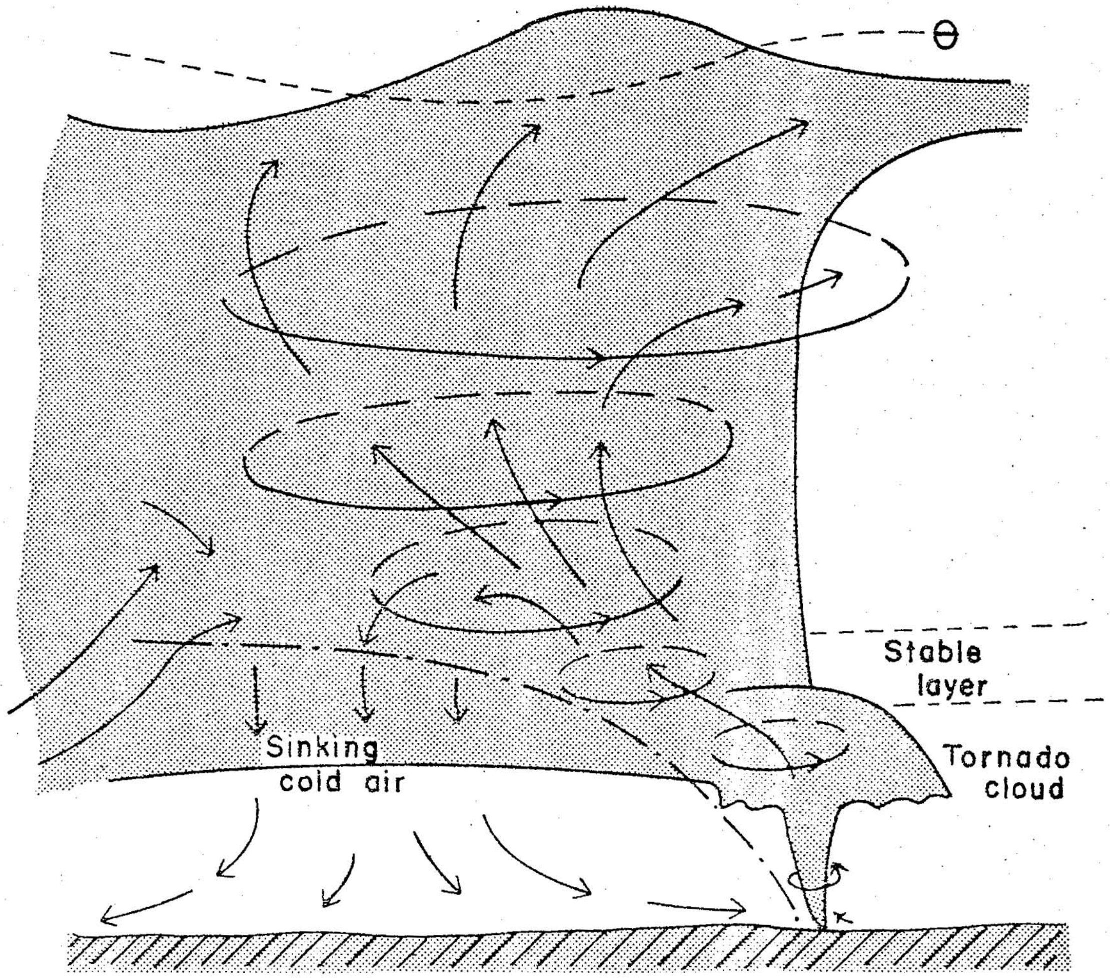

Figure 7.— Sketch, looking at right-hand end of a tornado thunderstorm, showing suggested flow patterns. Arrows indicate vertical flow and also horizontal outspreading both at high levels of the cloud and near the ground in the cold air. Ellipses represent organized vortical flow in approximately horizontal planes. θ is an isopleth of potential temperature above the cloud portion of the vortex.

7. THERMAL STABILITY IN AND AROUND THE VORTEX

With the suggested structure of the vortex, and the apparent need for convection through it from the warm moist lower air up to the region of intense divergence aloft, it seems necessary that in general there be vertical thermal stability at the top of the tornado cloud (defined here as the cloud from which the visible tornado extends downward) as shown in figure 7. Otherwise, the upward motion would tend to break through vertically and sever the connection of the low-level vortex from the upper one, causing the lower vortex to lose an important part of its energy source. There is observational evidence that a breakthrough from the tornado cloud directly upward does sometimes occur, and when it does the sudden uprush of air may cause a momentary intensification of the visible tornado. In such a case, the visible tornado is likely to weaken thereafter (because of its separation from the upper divergence field), unless changes aloft again bring the upper portion of the vortex into proper relationship to the thunderstorm. Often the tornado cloud appears to be quite flat and not very deep, except for short upward protuberances. According to the model presented here, the evacuation of air from this cloud is mostly into the right-hand side of the thunderstorm where in part it displaces sinking cold air and in part continues upward into the middle and upper right-hand side of the thunderstorm.

The suggested requirement for stability at the top of the tornado cloud implies that the air above it, which is presumed to come from the rear of the thunderstorm, is potentially warmer than air at the top of the tornado cloud itself, yet dry enough that it can be cooled by evaporation sufficiently to sink to the ground. Perhaps it is significant that these suggested criteria do not indicate direct association of the tornado with a cold front. But rather, in cases where the accompanying thunderstorm happens to be close to the cold front, it suggests that it is air from above the cold front surface which both streams forward around the thunderstorm aloft and sinks by evaporational cooling to form the cold air mass of the thunderstorm.

This idea is consistent with the frequently observed occurrence of tornadoes ahead of the cold front, where also the veering of winds with height (warm advection) is likely to be better developed. In the United States, tornadoes are observed frequently ahead of the cold front, by distances ranging up to a few hundred kilometers. While vertical stability at the top of the tornado cloud appears necessary to development of the tornado in its earlier stages, it is possible that at later stages the vortex sometimes becomes surrounded by colder air aloft, in which case the upper vortex must take on a somewhat different form.

Once the tornado vortex is well developed, its close contact with the right-hand end of the thunderstorm should permit rain and hail to be drawn into the outer portion of the vortex, at least in some cases. That this actually happens seems to be indicated by radar pictures (for example, Huff, Hiser, and Bigler, 1954) which show a hook-like echo that tends to form into a ring in the vicinity of the tornado. The outer diameter of this ring may be several kilometers and the inner clear area (area of no echo) at least two or three kilometers, perhaps more. It is unlikely that precipitation can ever penetrate the central portion of the tornado vortex, because of strong upward motion in the center and because particles of rain-drop or hail size would tend to be thrown outward by centrifugal action.

The outer portion of the vortex should, at least at some levels, be composed of the dryer environmental air because dynamic stability of the vortex would prevent mixing of the rising warm moist inner core with the outer portions of the vortex. Thus the falling precipitation should by evaporation tend to produce a ring of colder air around the outer portions of the vortex; the potential temperature of this air may well be lower than that of the surface warm air. This ring of cold air should be continuous with and become a part of the cold air mass of the thunderstorm, though the circulation and movement of the vortex may tend to carry the ring forward and aloft over the lower warm moist air.

Because of its temperature, the ring of cold air must in general be in the process of sinking. If the vortex were stationary, the colder air would sink to the ground and choke off the supply of warm moist air, tending to destroy the vortex. However, if the vortex is moving, the cold air which reaches its forward portion aloft need not be sinking fast enough to reach the ground before the vortex has passed, thus not preventing inflow of low-level warm moist air on the forward side but at the same time contributing to the energy of the vortex by increasing the buoyancy of the rising warm air.

Showalter (1943) in discussing the origin of tornadoes suggested that hail, thrown outward from the higher levels of the thunderstorm and falling through surrounding undisturbed air, would by evaporation and melting produce marked instability in the lower warm moist air provided the wet-bulb temperature decreases markedly with height just above the moist layer, and the $0^{\circ}C$. wet-bulb temperature be not far above the top of the lower moist air. He assumed the origin of the tornado to be primarily thermodynamic and that such instability was sufficient to produce the tornado; also, that there was a ceiling to convection in the tornado not far above the top of the moist layer. In the model presented here evaporation aloft is considered important in adding to the energy of the tornado, but not as a primary factor in causing it except in the indirect sense that evaporational cooling is necessary to the mechanics of the accompanying thunderstorm and as rain-cooled air plays a part in producing the upper divergence field at the right-hand end of the thunderstorm.

8. PRESSURE AND TEMPERATURE IN THE VORTEX

The strongest winds in a tornado have been estimated up to 200 m./sec. and even greater. Taking that value, we may estimate the depression of the pressure in the center. For air flowing horizontally toward lower pressure we may write on the basis of conservation of energy, neglecting the effect of friction and of movement of the pressure system

$$\Delta\left(\frac{V^2}{2}\right) = -\frac{\Delta p}{\bar{\rho}}, \qquad (1)$$

where V is the wind speed, p the pressure, $\bar{\rho}$ the mean density, $\Delta(V^2/2)$ the gain of kinetic energy per unit mass, and $-\Delta p/\bar{\rho}$ its loss of potential energy. Taking $V_0 = 0$ and $V = 200$ m./sec., $\bar{\rho} = 10^{-3}$ gram cm.$^{-3}$ we obtain for $-\Delta p$ a value of 200 mb. This value is comparable with the greatest pressure drops that have been reported (Flora, 1953, pp. 25-28). The work done by the pressure gradient force is partly lost through friction. However, air approaching the center of the vortex from its rear will move for a relatively longer time toward lower pressure, because the pressure field tends to move away from it, so that except for reduction by friction, speeds in excess of that indicated by the value of $-\Delta p$ would be possible in the right rear portion of the tornado.

The tornado exists for an appreciable period of time, frequently a half-hour and occasionally several hours. This means that through most of the life of the tornado the pressure in its center cannot be changing very rapidly except for possible short-period oscillations; in any case the pressure is stationary at the instant of minimum central pressure. It can be shown that whenever the central pressure is not changing, even though there are vertical accelerations the upward and downward accelerations are largely compensating, and the net effect on the pressure at the ground must be small. This net effect can be of the order of a few millibars at most.

Viscous forces have a possible effect on pressure distribution, as a result of non-linear horizontal variation of vertical velocity, and act in such a way as to tend to <u>increase</u> the surface pressure beneath a center of maximum upward velocity. Consequently, the viscous forces will if anything tend to <u>lessen</u> the pressure difference between the tornado and its surroundings. However, the effect of viscous forces including those of eddy viscosity appear to be negligible, and it seems reasonable to conclude that <u>the pressure at the ground in the center of the tornado at the instant of minimum pressure is largely hydrostatic.</u> Probably the same is true to a high degree of approximation even when the pressure is changing.

In any plane perpendicular to the vortex, whether the plane be horizontal or not, the average pressure-gradient force directed toward the center of the vortex is balanced by the centrifugal force of the rotation in that plane except where there are radial accelerations such as those caused by friction at the ground. As a result, the pressure deficit in the center is largely maintained by centrifugal force whether or not the vortex is vertical. If some portion of the vortex is horizontal, the buoyancy cannot accelerate the flow along its axis in that portion of the vortex. Hence in an equilibrium state (pressure not changing with time) there cannot be any large pressure gradient along the horizontal axis. Then, the pressure along the axis must vary essentially with respect to the vertical component of the axis so that the hydrostatic approximation should be a reasonable assumption even when the vortex is tilted away from the vertical.

It is of interest to determine the excess of mean virtual temperature, averaged with respect to height in the center of the vortex, that is necessary to account for the depression of central pressure in the tornado at the ground for any assumed depth of the vortex. Letting subscript 1 represent conditions in the center of the vortex, and subscript 2 those of the environment, it follows from the hydrostatic equation, combined with the equation of state, that

$$\ln\left(\frac{p_2}{p_1}\right) = \left(\frac{g}{R}\right)\left(\frac{1}{T_{m1}} - \frac{1}{T_{m2}}\right)Z,$$

where p is the surface pressure, g the acceleration of gravity, R the gas constant for one gram of air, Z the depth of the layer, and T_m the mean virtual temperature of the air column. The results of computations are shown in Table 1, where the indicated lowering of pressure (Δp) is computed from the given assumed wind speed in the center according to the energy relationship

$$\Delta\left(\frac{V^2}{2}\right) = -\left(\frac{\Delta p}{\bar{\rho}}\right),$$

taking $\bar{\rho}$ as 10^{-3} gm./cm.^{3}

Table 1 - Values of $(T_{m1} - T_{m2})$, the excess of mean virtual temperature in the center of the vortex required to account for a given Δp for various values of height Z. Δp is the pressure difference needed for generation of the specified values of V, the wind speed in the center, computed on the assumption that surrounding wind speeds are zero.

Z (km.)		10	20	30	40
Assumed T_{m2} (°K.)		250	235	230	230
V (m./sec.)	Δp (mb.)				
200	200	49°C.	20°C.	12°C.	9°C.
150	112.5	24	10	6	5
100	50	10	4	3	2
50	12.5	2	1	1	1/2

This table suggests the possibility that in the most extreme case the vortex may extend upward as high as 40 km. The only other possible interpretations of Table 1, aside from the remote possibility of some unknown factor, are either that there is some means of creating large temperature differences, or that the assumed depression of 200 mb. never occurs. In considering the meaning of the temperature differences, note should be taken of the fact that adiabatic expansion decreases the temperature at the center, especially in the lower portion of the vortex. On the other hand in at least the most severe tornadoes it is probable that air subsides in the center much of the way, though probably not all the way, to the ground. For this reason, the vortex may extend considerably above the level reached by ascending air in its surroundings--well into the clear air above the clouds.

9. GENERATION OF STRONG WINDS AND FORMATION OF THE VISIBLE TORNADO.

In the discussion so far, a physical model has been proposed only for the main features causing formation of the initial large-scale vortex of several kilometers radius. Although this vortex is taken as a required initial condition, it is necessary to consider the further requirements for formation of a more concentrated vortex. Frictionally-induced vertical motion combined with strong buoyancy of the rising air, and possibly a particular vertical distribution of temperature and moisture through the lower moist layer, seem to be the necessary physical factors permitting or causing formation of the more concentrated vortex. A satisfactory model of the intensification of the vortex must provide some means for horizontal contraction of individual rising air parcels so as to increase their vorticity, and for accumulation of energy in the vortex.

Probably the Rankine combined vortex is as good an approximation as can be justifiably assumed for the low-level tornado vortex, including the initial large-scale vortex which is assumed to form during a short initial period of adjustment following the first pressure fall superimposed from above. In the Rankine combined vortex there is a radius of maximum tangential speed inside which v/r = constant and outside which vr = constant. That is, there is a central portion rotating as a solid and an outer irrotational region having no definite outer boundary. If v_m is taken as the maximum tangential speed and r_m its radius, both assumed constant, it follows from the definition of the Rankine vortex that for the inner vortex, $v = v_m r/r_m$, and for the outer region, $v = v_m r_m/r$. This model of a vortex is not quite realistic but will serve sufficiently well for further deductions where only the order of magnitude is significant.

While it has been shown in Table 1 that certain extreme low-level wind speeds are compatible in magnitude with certain values of Δp, our discussion of the actual mechanics of the vortex has so far accounted for a depression of only a few millibars in pressure, and for wind speeds of the order of magnitude of the environmental winds. Before proceeding to the mechanics of the further intensification of the vortex, we will show by application of the Rankine vortex that these initial winds and pressure differences are of a consistent order of magnitude. Neglecting friction and accelerations, the pressure gradient force in a rotating system is balanced by the centrifugal force, that is,

$$\frac{v^2}{r} = \left(\frac{1}{\rho}\right)\frac{\partial p}{\partial r},$$

where v is the tangential speed. By substitution for v from the Rankine combined vortex we obtain, for the pressure difference between the center and some assumed outer radius r_n,

$$\Delta p = \bar{\rho}\left(\frac{v_m}{r_m}\right)^2 \int_0^{r_m} r\,dr + \bar{\rho}(v_m r_m)^2 \int_{r_m}^{r_n} \frac{dr}{r^3}, \qquad (2)$$

$$\text{or } \Delta p = \bar{\rho} v_m{}^2 \left[\left(\frac{1}{r_m{}^2}\right)\left(\frac{r_m{}^2}{2}\right) - \left(\frac{r_m{}^2}{2}\right)\left(\frac{1}{r_n{}^2} - \frac{1}{r_m{}^2}\right)\right]$$

$$= \bar{\rho} v_m{}^2 \left(1 - \frac{r_m{}^2}{2r_n{}^2}\right).$$

Taking v_m = 20 m./sec., r_m = 2 km., r_n = 5 km., $\bar{\rho} = 10^{-3}$ gm./cm.^{3},

$$\Delta p = 3.7 \text{ mb.}$$

or, if v_m = 25 m./sec., r_m = 1 km., r_n = 5 km.,

$$\Delta p = 6.1 \text{ mb.}$$

All these values of v_m, r_m, r_n and the computed Δp appear to be of a reasonable order of magnitude for the initial vortex.

As soon as the initial low-level vortex is formed, there must be frictionally-induced inflow at every point near the ground. Suppose as a rough approximation we take the radial inflow to be directly proportional to the tangential component of wind speed. While this approximation is not the best, it is unlikely that greater refinement will be useful for the present purpose. The divergence in a symmetrical circular vortex, expressed in polar coordinates, is

$$\text{Div } \mathbf{V} = \frac{\partial v_r}{\partial r} + \frac{v_r}{r},$$

where v_r is the radial velocity dr/dt. The divergence associated with the assumed frictional effect (subscript F) would then in general be

$$\text{Div}_F \mathbf{V} = -\text{ Const. x } \left(\frac{\partial v}{\partial r} + \frac{v}{r}\right),$$

where v is the tangential speed. If we assume further that the tangential speed is everywhere proportional to that which would exist in the absence of frictional effect, and substitute for v in terms of the Rankine combined vortex, we obtain

$$\left.\begin{aligned} \mathrm{Div}_F V &= -\text{Const. x}\left(\frac{v_m}{r_m} + \frac{v_m}{r_m}\right) = -\text{Const.}\,[\text{inner vortex}] \\ \mathrm{Div}_F V &= -\text{Const. x}\left(\frac{v_m r_m}{r^2} - \frac{v_m r_m}{r^2}\right) = 0\,[\text{outer region}] \end{aligned}\right\} \quad (3)$$

Thus, for the given model and assumed frictional effect, frictionally caused convergence is confined to the inner vortex and is uniform throughout that area. Such a distribution of convergence will not hold exactly in nature, but undoubtedly the approximation is sufficiently reasonable to permit the conclusion that ground friction produces a central region of strong horizontal convergence and therefore a central core of rapidly rising air.

While friction dissipates kinetic energy and provides eventually for equilibrium between the formation and dissipation of kinetic energy, it also provides for a central region in which energy of buoyancy becomes available if the rising air is sufficiently warm compared with the surrounding air. In the visible part of the tornado, most of the warming is evidently through condensation of the rising air. For any individual parcel, the magnitude of heating is largely independent of the frictional effect. If, as a result of friction, there is an increase in the rate at which buoyant air is being supplied to the central core, and the energy of buoyancy of each rising air parcel continues to exceed the dissipative effect of friction, the kinetic energy of the vortex must continue to increase provided the energy of buoyancy can be utilized. The effect of friction is likely an important reason why tornadoes over land are generally more severe than similar phenomena over the ocean.

Once a column of rising air is established, its buoyancy will accelerate the upward flow and tend to develop a field of vertical motion in which the upward speeds increase with height. Each air parcel will then be stretched vertically as it ascends, and it must therefore at the same time undergo horizontal contraction. Any ascending horizontal ring of air parcels rotating about the center of the vortex will then contract as it proceeds upward. By the law of conservation of angular momentum, a contracting ring must increase its linear tangential speed and work must therefore be done upon it. However, in an irrotational field (vorticity = 0) each contracting ring will merely replace another having the same tangential speed and there will be no net change in the velocity field. This condition exists in the outer region of the Rankine combined vortex, so that such horizontal contraction of individual rings of air in the outer region should not contribute to a net increase in energy of the system. But in the inner vortex, where the linear tangential speeds decrease toward the center, any contraction will cause a marked increase in speed. The contraction, on the other hand, is opposed by the increase of centrifugal force and thus the process can proceed only as sufficient vertical stretching effect takes place. In this manner a portion, apparently a very large portion, of the energy of buoyancy must be transferred to the energy of rotation rather than to upward speed.

With increased speed of rotation, the central pressure of the vortex must decrease. Since by the process proposed here the increase of rotation depends on buoyancy, the effect must operate toward reduction of the central pressure at all levels where the rising air is buoyant. The magnitude of the effect may vary from level to level, but because the vertical pressure distribution must remain essentially hydrostatic, the total effect should be felt almost immediately at all levels in the vortex. The pressure must fall also above the levels of buoyancy and of upward motion, where horizontal convergence and downward motion accompanied by adiabatic warming should result. The process is thus one of coincidentally increasing the intensity of the vortex at any level and of increasing the height to which it extends. Decreasing central pressure in the lower levels will tend to increase horizontal acceleration of air into the vortex (an isallobaric effect) and thus the process of intensification should be unstable until frictional dissipation becomes a more effective brake at the higher surface wind speeds. The actual work of increasing the speed of rotation in the tornado is done by the horizontal pressure forces, and the source of energy is the solenoidal field, but for convenience the buoyancy force will be considered here as the means of estimating the order of magnitude of the energy that goes into strengthening the vortex.

Each parcel of air ascending in the vortex therefore contributes to an increase in the kinetic energy of the rotation as long as the energy provided by the buoyancy forces exceeds the dissipative effect of friction. This process also provides for contraction of the inner vortex, and can account for full development of the powerful vortex if it can be shown to be of the proper order of magnitude.

In the following computations, the work done by buoyancy forces in the central core will be compared with the work required to contract this core to a smaller radius, in which the kinetic energy is greater by virtue of the higher speeds generated as a result of such contraction. The acceleration due to buoyancy of the central column is $g\Delta T/T$, where ΔT is the excess of temperature in the central column over that of the environment at the same pressure. Since work (W) is defined as force (or acceleration x mass) x distance travelled, it follows that the work per unit time

$$\frac{dW}{dt} = \text{acceleration} \cdot \text{density} \cdot \text{volume} \cdot \text{speed}.$$

If we consider the upward acceleration and speed in a vertical cylinder in which the average density is $\bar{\rho}$, the height h, and the radius r_m,

$$\frac{dW}{dt} = g\frac{\Delta T}{T} \cdot \bar{\rho} \cdot (\pi r_m{}^2 h) \cdot w;$$

$$\frac{dW}{dt} = \pi\bar{\rho}hr_m{}^2 gw\,\frac{\Delta T}{T}\,. \qquad (4)$$

To obtain a numerical estimate of the rate of energy release the following values are assigned: $\bar{\rho} = 10^{-3}$ gm./cm.^{3}, h = 1 km., r_m = 0.5 km., w = 20 m./sec., $\Delta T = 3^0$ C., T = 300° K. Then

$$\frac{dW}{dt} = \pi \cdot 10^{-3} \cdot 10^5 \cdot 25 \cdot 10^8 \cdot 10^3 \cdot 2 \cdot 10^3 \cdot (3/300) \text{ cgs units}$$

$$= \frac{\pi}{2}\,10^{16} \text{erg/sec.} \qquad (5)$$

is the rate of energy release in the central core in a column 1 km. high.

In making the preceding computation of energy of buoyancy, only an averagevalue was chosen. Probably, however, a factor of importance in permitting the formation of the tornado vortex is the distribution of buoyancy with height, and consequently also the stretching effect, through the lower warm moist layer. Carr (1954) empirically determined certain distributions of temperature and humidity through the lower moist layers of the atmosphere that tend to accompany or precede tornadoes. These appear to support a hypothesis that the buoyancy of a rising air parcel must increase with height in order for tornadoes to occur.

We cannot say how much of the energy of buoyancy goes into the energy of the vortex, though it must be a substantial portion because the vertical stretching effect on an ascending column in the absence of the resisting force of a vortex can be shown to be sufficient to cause it to expand vertically by several times its initial height. The resistance of the vortex to horizontal contraction will permit only a small fraction of the stretching to actually take place. Here we disregard the expansion caused by decreasing density with height; this expansion does work against the environment at the expense of internal energy but without necessarily affecting the contraction of the vortex.

If one-half the energy of buoyancy (equation 5) is assumed to go into the energy of the vortex, and we consider the effect over a period of ten minutes:

Work done by buoyancy forces

$$= 1/2(\tfrac{\pi}{2})\cdot 10^{16} \text{ erg/sec. x } 600 \text{ sec.}$$

$$= 6(\tfrac{\pi}{4})\cdot 10^{18} \text{ erg per 10 min.} \tag{6}$$

This effect will spread through a greater depth than that over which it is released, but there will also be buoyancy and additional energy released through a greater depth, so that we may reasonably consider this amount of energy to equal that taken up by the lower kilometer of the vortex; actually we assume for convenience that the depth increases by whatever amount is necessary to permit horizontal contraction.

The kinetic energy of a horizontal slice of a cylindrical vortex having <u>unit mass</u> is

$$\int_0^r \frac{V^2}{2}\cdot\rho 2\pi r\Delta h dr.$$

The volume of this slice, equivalent to the specific volume since it has unit mass, is $\pi r_m^2\Delta h$, defining the thickness Δh. Here r_m is the radius of the cylinder in solid rotation (v/r = constant). By substitution from $v/r = v_m/r_m$ and from the above expression for the specific volume into the preceding expression, it is found that

$$\text{KE (slice of unit mass)} = \frac{V_m^{\,2}}{r_m^{\,4}}\int_0^{r_m} r^3 dr = \frac{V_m^{\,2}}{4} \tag{7}$$

For the work required to contract the vortex and thus to increase its kinetic energy, we may write by use of (7)

$$\text{Work done on vortex} = \text{mass x } \left(\frac{V_{mf}^{\,2}}{4} - \frac{V_{mo}^{\,2}}{4}\right) \tag{8}$$

where subscript f denotes final and subscript o an initial state. If r_{mo} = 0.5 km., the mass $\pi r_{mo}^{\,2} h_o\bar{\rho}$ of a column of 1-km. depth and average density 10^{-3} gm./cm.^{3} is found to be $\pi\cdot(10^5/2)^2\cdot 10^5\cdot 10^{-3}$ gm. = $(\pi/4)\cdot 10^{12}$ gm. Assigning a value of 20 m./sec. for v_{mo} we find from (8) that

$$\text{Work done on vortex} = \frac{\pi}{4}\cdot 10^{12}\left(\frac{V_{mf}^{\,2}}{4} - 10^6\right) \text{ ergs.}$$

Equating this to the work assumed available from buoyancy forces (equation 6), it is found that

$$V_{mf} = 53 \text{ m./sec.}$$

With conservation of circulation in the outer contracting ring, $v_{mf}\, r_{mf} = v_{mo}\, r_{mo}$, giving

$$r_{mf} = 0.19 \text{ km.} \approx 2/5 \text{ initial radius.}$$

This comparison suggests that the energy available from buoyancy forces during a 10-minute period is sufficient to cause a contraction of the initial vortex to less than half its initial radius, leading to a significant increase of the winds in the vortex. In making this computation we have considered only the increase of energy in the inner vortex (where v/r is assumed constant). There will also be an increase of kinetic energy in the adjacent outer region (where vr is assumed constant), and this must be of at least the same order of magnitude as the increase of energy in the inner vortex. Note, however, that only half the available energy production has been made use of in the above computation.

In the above we have not considered the possible work which might be done on the lower vortex by the initial vortex aloft, considered as the primary cause of the parent micro-low. The fact that under the assumed conditions such an upper vortex must exist, and must be a region from which a large volume of air may be exhausted, means that energy additional to that of buoyancy will be supplied to the vortex because the pressure will tend to be maintained at a slightly lower than hydrostatic value as computed from the ground upward. We can make an estimate of this effect; a depression of 5 mb. appears to be not unreasonable (compare earlier estimate in Section 5), though this value will be reduced if air is fed rapidly from below as the vortex develops. Assuming the pressure reduction to remain at 3 mb.at a height of 6 km., we compute the effect on vertical acceleration:

$$\overline{\left(\frac{dw}{dt}\right)} = \left(\frac{1}{\rho}\right)\left(\frac{\partial p}{\partial z}\right) \approx \left(\frac{3000}{10^{-3}\cdot 6\cdot 10^{5}}\right) = 5 \text{ cm./sec.}^2$$

This is one-half the value assumed for the buoyancy force and is therefore appreciable, the condition for its realization being a breakthrough -- formation of a channel for upward flow into that region -- such as becomes established with the strengthening of the lower-level vortex by buoyancy forces.

From the estimates of energy available and assuming that only about one-half goes into the energy of the vortex because of frictional loss, it is estimated that the inner vortex may contract to one-half its diameter in a period of the order of 10 minutes, and that this approximate rate of development will be maintained up to the final development of the tornado because as the tornado develops and greater amounts of energy become necessary, the vertical speed will also increase so as to release greater amounts of energy of buoyancy. <u>A total time period for development of the order of</u>

one-half to one hour appears reasonable; probably in some cases it is longer. It is perhaps significant that because the tornado tends to occur with the elongated squall-line type thunderstorm, it is probably in general associated with a thunderstorm having longer than the average life.

The process of intensification of the vortex pictured above requires that ordinarily the center of the column first become visible just below the cloud base, because the condensation level is initially at the cloud base and is lowered progressively from that level as the pressure in the center of the column becomes lower. The pressure at the base of the visible column (representing the pressure in the center of the vortex at that level) should be at least approximately the same as that of the cloud base from which it descends. The visible tornado therefore moves downward from the cloud, and upon dissipation it moves upward, as is commonly observed.

10. DIRECTION OF ROTATION

The tornado normally has cyclonic rotation, consistent with the fact that it occurs where winds veer with height. The mechanism outlined here does not provide for tornadoes in which the micro-low rotates anticyclonically, except in the unlikely case of thunderstorms occurring where winds back with height. On the other hand, more than one tornado may occur in one parent micro-low; this is to be expected if there are departures from symmetry in the parent vortex, such as are especially likely in a moving system, so that convective columns break through irregularly. Once a tornado with cyclonic rotation has developed in an inner region, there must be considerable anticyclonic wind shear over the portion of the parent vortex surrounding the tornado. It is conceivable that with rapid irregular accelerations, the anticyclonic shear may occasionally exceed the cyclonic curvature so that there is not only local anticyclonic vorticity but also dynamic instability and therefore the possibility of rapid local convergence. This could result in an anticyclonic whirl; in fact there seems to be no particular inconsistency in having tornadoes of opposite rotation within the same parent vortex, though the first tornado must always rotate in the same sense as the parent vortex and, in terms of frequency, the number of tornadoes rotating the same as the parent vortex must be predominant.

11. SYNOPTIC ASPECTS

In terms of data normally available in synoptic practice, or which can be readily computed, the model presented here suggests that the formation of tornadoes is primarily a function of the combined magnitude of geostrophic warm advection and vertical thermal instability. The magnitude of geostrophic warm advection is roughly proportional to the amount of veering of wind with height. The association of warm advection and instability with tornadoes is already well known from synoptic experience. Warm advection, in addition to being directly favorable to tornado development at the right-hand end of an elongated thunderstorm, is usually accompanied by general upward motion over the region where it occurs. The upward motion (which implies vertical stretching in the lower troposphere) tends to produce or increase conditional instability, and because of associated low-level convergence the region must also be one of increasing accumulation of total precipitable moisture, at least until some time after precipitation has developed. The distribution with height of both warm advection and thermal instability also appear to be important.

Another feature presumably essential to tornado formation is convective instability, a vertical lapse rate of wet-bulb temperature in the environment generally greater than the saturation adiabatic. This is necessary in order that evaporation from falling rain will produce subsidence. Subsiding rain-cooled air is almost certainly essential to formation and maintenance of the squall-line thunderstorm and, in terms of the present model, its existence in low levels is necessary to formation of the tornado.

Tornadoes are sometimes isolated and sometimes widespread. This means that local conditions necessary and sufficient for their formation, whatever they are, may or may not exist over an area large enough to be detected in the synoptic network. It follows that observations from a closer network, or some other means of more accurately determining the areal distribution of pertinent elements, will be necessary before isolated tornado occurrences can be anticipated with reasonable accuracy. Similarly, carefully planned three-dimensional observations from a close network will be required to test models or theories of formation of the tornado and to extend our knowledge of its mechanics.

A further aspect not especially discussed in this paper is that tornadoes often occur within a local low-pressure area of the order of 100-200 km. diameter; this low may be a secondary in a larger cyclone, a frontal wave, or a completely isolated low often difficult to detect in the observational network. Probably, because of its relative size and the fact that it exists for several hours up to a half day or longer, it is the smallest scale phenomenon associated with the tornado for which the earth's rotation is of direct significance. The low-level frictional convergence accompanying it should contribute some to the energy of the tornado micro-low, but perhaps its most important function is to contribute to the energy of the accompanying thunderstorm by producing general upward motion in the region.

The tendency for thunderstorms to occur in regions of maximum low-level warm advection was pointed out by Means (1944), and a similar tendency for tornadoes to occur in regions of warm advection has been established by synoptic experience. It has recently been shown by Gilman (1954) and associates that where the Laplacian of temperature advection, $\nabla^2(\mathbf{V}\cdot\nabla T)$, indicates a region of local low-level warming, that region is a favorable place for vertical motion. These facts indicate that tornadoes and much thunderstorm activity occur in a local region where the prevailing vertical motion is upward for dynamic reasons. Local low-level horizontal convergence associated with the general upward motion will tend to form, according to the theorem of conservation of potential vorticity, a local low-pressure center that in many cases will be of smaller scale than the cyclone. There is therefore undoubtedly a close connection between local low-level warm advection and the formation of small secondary low-pressure centers.

There seems to be no reason why the conditions which cause tornadoes over land cannot also occur over the oceans, except that there is modification by the difference in frictional effect, and also because rain-cooled air over an ocean surface will warm more rapidly than over land. Probably, therefore, the same distinction can be made over the ocean as between tornadoes and dustwhirls over land, some waterspouts being of the tornado type while others, less intense and more frequent, are caused by the existence of an air mass cooler than the water surface.

12. CONCLUSION

The primary condition necessary to formation of the tornado is considered to be the simultaneous development of upper divergence and cyclonic vorticity around the right-hand end of the elongated or squall-line type thunderstorm, caused by upward movement of air from low levels through an environment in which the winds veer with height. Essential to realization of actual net divergence aloft are the separation of uplifted air from the updraft of the storm, the sinking of rain-cooled air beneath the region of upper divergence, and the subsidence of thermally stable air at some level above the region of upper divergence. For convection to develop in the vortex it must extend into warm air at the ground. After the initial vortex forms, and convection is established through it from the ground upward, buoyancy causes each air parcel to be stretched vertically and contracted horizontally so as to increase the kinetic energy of the vortex, the result being an accumulation of energy in the vortex with each new ascending air parcel and eventual formation of the severe tornado.

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